Virtual Compton Scattering and Generalized Polarizabilities of the Nucleon in Heavy Baryon Chiral Perturbation Theory

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The spin-independent part of the virtual Compton scattering (VCS) amplitude from the nucleon is calculated within the framework of heavy baryon chiral perturbation theory (HBChPT). The calculation is performed to third order in external momenta according to chiral power counting. The relation of the tree-level amplitudes to what is expected from the low-energy theorem is discussed. We relate the one-loop results to the structure coefficients of a low-energy expansion for the model-dependent part of the VCS amplitude recently defined by Fearing and Scherer. Finally we discuss the connection of our results with the generalized polarizabilities of the nucleon defined by Guichon, Liu and Thomas.

1 Introduction

With the growing number of experimental proposals [1] seeking to investigate virtual Compton scattering (VCS) off a baryonic target there has been correspondingly increased activity within the theoretical community examining VCS from the nucleon [2,3,4,5,6] as well as from nuclei [7] in recent months. The primary objective of the real Compton scattering experiments involving the nucleon has traditionally been the determination of the nucleon electric and magnetic polarizabilities — α_0 and β_0.[8] On the theoretical side these polarizabilities have been calculated within various models (see, *e.g.*, the review articles by Holstein [9] and L'vov [10]). The pioneering work on the generalization of the electromagnetic polarizabilities of the nucleon to the VCS case [2] (here we only refer to the spin-averaged amplitude),

$$\gamma^*(\varepsilon^\mu, q^\mu) + N(p_i^\mu) \to \gamma(\varepsilon'^\mu, q'^\mu) + N(p_f^\mu) \quad ({q'}^2 = 0\,,\ q^2 = -Q^2 < 0)\,, \quad (1)$$

made a first prediction for the generalized electric and magnetic polarizability as a function of the initial photon three-momentum, $\alpha\,(|\,\vec{q}\,|)$ and $\beta\,(|\,\vec{q}\,|)$,

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within the framework of a non-relativistic constituent quark model [2]. Subsequently, an effective Lagrangian approach [3], a refined version of the quark model[5] and the linear sigma model[6] have been used to make predictions for the generalized polarizabilities. Herein we will add the prediction of chiral perturbation theory (ChPT), the effective description of quantum chromodynamics in the nonperturbative energy regime with nucleons and pions as physical degrees of freedom [11,12,13], to third order in the external momenta — $O\left(p^3\right)$, generalizing the work of Bernard et al. [12,14] on the real Compton process to VCS. In order to achieve this, it is necessary to decompose the invariant VCS amplitude $\mathcal{M}_{VCS}$ into a structure-dependent part, which is regular and consists of contributions from one-particle irreducible Feynman diagrams (for the spin-independent part of the VCS amplitude these are the same loop diagrams as in Bernard et al. [12]), and a singular structure- or model-independent part, which is generated by s- and u-channel Born diagrams, together with the seagull graph required by gauge invariance. For this decomposition we follow the convention of Guichon et al. [2] and Scherer et al. [4], where the Born contribution is calculated in terms of Dirac and Pauli form factors, F_1 and F_2. Recently, a general low-energy parametrization of the structure-dependent part of the VCS amplitude on a spin 0 target has been developed by Fearing and Scherer [15]. Since it has been shown by Bernabéu and Tarrach[16] that the form of the *spin-independent* part of the VCS amplitude for a spin $\frac{1}{2}$ target is the same as for a spin 0 target, we can apply this parametrization to our calculation in order to determine the nine structure coefficients in the low-energy expansion to fourth order in the four-momenta of the initial and final state photons. Finally we will relate these structure coefficients to the generalized polarizabilities of the multipole expansion by Guichon et al. [2]. A more comprehensive discussion of the material presented in this contribution can be found in Hemmert et al. [17].

2 Chiral Calculation of the spin-independent VCS Amplitudes

The invariant amplitude for VCS can be decomposed into a transverse and a longitudinal part and, using current conservation, can be written as [4]

$$\mathcal{M}_{VCS} = -ie^2\varepsilon_\mu M^\mu = -ie^2\varepsilon_\mu\varepsilon_\nu'^* M^{\mu\nu} = ie^2\left(\vec{\varepsilon}_T\cdot\vec{M}_T + \frac{q^2}{\omega^2}\varepsilon_z M_z\right). \quad (2)$$

Since we will restrict our discussion to the spin-averaged component of $\mathcal{M}_{VCS}$ in the following (the analogous spin-dependent calculation will be the subject of a future publication), the invariant amplitude can be parametrized in terms

of three independent structures,

$$\mathcal{M}_{VCS}^{non-spin} = ie^2 \left[\vec{\varepsilon}'^* \cdot \vec{\varepsilon}_T A_1 + \vec{\varepsilon}'^* \cdot \hat{q} \vec{\varepsilon}_T \cdot \hat{q}' A_2 + \vec{\varepsilon}'^* \cdot \hat{q} \frac{q^2}{\omega^2} \varepsilon_z A_9 \right] , \tag{3}$$

two of which (A_1 and A_2) are purely transverse and one (A_9) which is longitudinal. A standard calculation of the s- and u-channel Born terms together with the seagull contribution with the Lagrangian of HBChPT (see, *e.g.*, Bernard et al. [12] and Ecker and Mojžiš [13]) yields the tree-level results

$$\begin{aligned}
A_1^{tree} &= -\frac{1}{2M} (1+\tau_3) , \\
A_2^{tree} &= \frac{1}{2M^2} (1+\tau_3) \mid \vec{q} \mid , \\
A_9^{tree} &= (1+\tau_3) \left(-\frac{1}{2M} + \frac{1}{2M^2} \mid \vec{q} \mid \cos\theta + \frac{1}{4M^2} \frac{\mid \vec{q} \mid^2}{\omega'} \right) ,
\end{aligned} \tag{4}$$

where M is the nucleon mass and τ_3 the isospin operator. We have written the results in terms of three independent kinematical quantities in the c.m. system — the energy of the outgoing photon ω', the three-momentum of the initial state photon $|\vec{q}|$ and the angle between the three-momenta of the initial and final state photons θ. We have used approximations for the initial photon energy ω and the Lorentz invariant quantities $t = (q-q')^2$ and q^2,

$$\begin{aligned}
\omega &= \omega' + O\left(r^2/M\right) , \\
t &= -\omega'^2 - \mid \vec{q} \mid^2 + 2\omega' \mid \vec{q} \mid \cos\theta + O\left(r^3/M\right) , \\
q^2 &= \omega'^2 - \mid \vec{q} \mid^2 + O\left(r^3/M\right) \qquad (r \in \{\omega', \mid \vec{q} \mid\}),
\end{aligned} \tag{5}$$

in order to be consistent with chiral power counting at $O\left(p^3\right)$, the chiral order to which our calculation applies. The structure-dependent part of the VCS amplitude is generated by nine diagrams [12]. The evaluation of the diagrams is straightforward but tedious. Using Eq. (5) and expanding the loop integrals in r/m_π, we obtain to order r^4: [b]

$$\begin{aligned}
A_1^{loop} = \frac{g_A^2}{F^2} \frac{1}{\pi m_\pi} \Bigg[& \frac{5}{96} \omega'^2 + \frac{1}{192} \omega' \mid \vec{q} \mid \cos\theta \\
& + \frac{17}{1920} \frac{1}{m_\pi^2} \omega'^4 + \frac{19}{1920} \frac{1}{m_\pi^2} \omega'^3 \mid \vec{q} \mid \cos\theta - \frac{1}{384} \frac{1}{m_\pi^2} \omega'^2 \mid \vec{q} \mid^2 \\
& - \frac{1}{320} \frac{1}{m_\pi^2} \omega'^2 \mid \vec{q} \mid^2 \cos^2\theta + \frac{1}{960} \frac{1}{m_\pi^2} \omega' \mid \vec{q} \mid^3 \cos\theta \Bigg] ,
\end{aligned} \tag{6}$$

[b] Here r is a generic small kinematic quantity such as $\omega', |\vec{q}|$.

$$
\begin{aligned}
A_2^{loop} &= \frac{g_A^2}{F^2}\frac{1}{\pi m_\pi}\left[-\frac{1}{192}\omega' \mid \vec{q} \mid - \frac{1}{384}\frac{1}{m_\pi^2}\omega'^3 \mid \vec{q} \mid \right. \\
&\left. + \frac{1}{320}\frac{1}{m_\pi^2}\omega'^2 \mid \vec{q} \mid^2 \cos\theta - \frac{1}{960}\frac{1}{m_\pi^2}\omega' \mid \vec{q} \mid^3 \right], \qquad (7) \\
A_9^{loop} &= \frac{g_A^2}{F^2}\frac{1}{\pi m_\pi}\left[\frac{5}{96}\omega'^2 + \frac{17}{1920}\frac{1}{m_\pi^2}\omega'^4 + \frac{7}{960}\frac{1}{m_\pi^2}\omega'^3 \mid \vec{q} \mid \cos\theta \right. \\
&\left. - \frac{7}{960}\frac{1}{m_\pi^2}\omega'^2 \mid \vec{q} \mid^2 \right]. \qquad (8)
\end{aligned}
$$

Note that the loop results for VCS from a proton or a neutron are identical, whereas the tree-level component vanishes for the neutron, because to $O\left(p^3\right)$ in the spin-averaged part of the amplitude only the photon coupling to the electric charge and not to the magnetic moment generates non-zero contributions.

3 Structure Coefficients and Generalized Polarizabilities

In Fearing and Scherer [15] a general low-energy parametrization of the structure-dependent VCS amplitude on a spin 0 target based on Lorentz and gauge invariance and the discrete symmetries has been generated to fourth order in the photon four-momenta. Using the results of Bernabéu and Tarrach [16] such a low-energy expansion also applies to the spin-averaged part of the VCS amplitude from a spin $\frac{1}{2}$ target. We have determined the *a priori* unknown structure constants in this expansion,

$$
\begin{aligned}
g_0 &= \frac{1}{192}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi}, & \tilde{c}_1 &= -\frac{11}{192}\frac{1}{8M^2}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi}, \\
g_{2a} &= \frac{1}{320}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, & g_{2b} &= -\frac{1}{960}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, \\
g_{2c} &= \frac{1}{1920}\frac{1}{16M^2}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, & c_3 &= -\frac{3}{1280}\frac{1}{8M^2}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, \\
\tilde{c}_{3a} &= \frac{1}{240}\frac{1}{8M^2}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, & \tilde{c}_{3b} &= -\frac{1}{256}\frac{1}{8M^2}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, \\
\tilde{c}_{3c} &= -\frac{9}{640}\frac{1}{128M^4}\frac{g_A^2}{F^2}\frac{1}{\pi m_\pi^3}, & & \qquad (9)
\end{aligned}
$$

where the notation is the same as in Fearing and Scherer [15]. Since our expansion of kinematical quantities, which is adequate for the application of HBChPT at $O(p^3)$ is different from the multipole expansion [2], it is not obvious how to relate the two approaches. However, starting from the general structure analysis [15]

and then applying Eq. (5), one finds relations between $\alpha\,(|\,\vec{q}\,|)$ and $\beta\,(|\,\vec{q}\,|)$, the generalized polarizabilities [2] $P^{(01,01)0}\,(|\,\vec{q}\,|)$ and $P^{(11,11)0}\,(|\,\vec{q}\,|)$, and the structure coefficients of the low-energy expansion [15]:

$$
\begin{aligned}
\alpha\,(|\,\vec{q}\,|) &= \alpha_0\left(1-\frac{7}{50}\frac{|\,\vec{q}\,|^2}{m_\pi^2}+O\left(\frac{|\,\vec{q}\,|^4}{m_\pi^4}\right)\right)=-\frac{e^2}{4\pi}\sqrt{\frac{3}{2}}P^{(01,01)0}\,(|\,\vec{q}\,|) \\
&= -\frac{e^2}{4\pi}(g_0+8M^2\tilde{c}_1-|\vec{q}|^2\,(g_{2b}+8M^2(c_3+\tilde{c}_{3b})) \\
&\quad +O((|\vec{q}\,|\,/m_\pi)^4))\,, \qquad (10) \\
\beta\,(|\vec{q}|) &= \beta_0\left(1+\frac{1}{5}\frac{|\vec{q}|^2}{m_\pi^2}+O\left(\frac{|\vec{q}|^4}{m_\pi^4}\right)\right)=-\frac{e^2}{4\pi}\sqrt{\frac{3}{8}}P^{(11,11)0}\,(|\vec{q}|) \\
&= \frac{e^2}{4\pi}(g_0-|\vec{q}|^2\,g_{2b}+O((|\vec{q}\,|\,/m_\pi)^4))\,. \qquad (11)
\end{aligned}
$$

These results agree with the corresponding terms of the linear sigma model calculation [6]. Moreover, it is worth mentioning that it is not possible to determine α from the transverse amplitude A_1, because the way in which we expand kinematical quantities introduces contributions from higher multipoles (see, e.g. the $\omega'^2\,|\,\vec{q}\,|^2\cos^2\theta$ term in Eq. (6)) and thus obscures the $L'=1$ part of the amplitude where we would expect $\alpha\,(|\,\vec{q}\,|)$. However, in the longitudinal amplitude (A_9) terms of higher multipolarity are absent, which allows us to extract $\alpha\,(|\,\vec{q}\,|)$. The third scalar polarizability in the multipole expansion [2], $\hat{P}^{(01,1)0}$, is $1/M$ suppressed and thus vanishes in our $O\left(p^3\right)$ calculation. However, a general, model-independent analysis of the spin-averaged part of the VCS amplitude [18] confirms the result of the sigma model calculation [6] that this polarizability can in fact be written as a linear combination of α and β. In comparison with the other calculations [2,3,5] our value for the electric polarizability of the proton, $\alpha_0=12.8\times10^{-4}\text{fm}^3$, is significantly larger, and for the magnetic polarizability of the proton our result, $\beta_0=1.3\times10^{-4}\text{fm}^3$, is smaller. The absolute values of the slopes $\frac{d}{d|\vec{q}|^2}\alpha\,(|\,\vec{q}\,|=0)$ and $\frac{d}{d|\vec{q}|^2}\beta\,(|\,\vec{q}\,|=0)$ are found to be considerably larger than in the other calculations, the slope of the magnetic polarizability even has a different sign. An experiment where both ω' and $|\,\vec{q}\,|$ are smaller than the pion mass should be able to distinguish between the different theoretical predictions. In this kinematical region the multipole expansion [2] to first order in the final state photon energy may not reliably parametrize the VCS amplitude, because, as has been shown above, contributions from higher multipoles enter the amplitudes at the same level as the slopes of the electromagnetic polarizabilities and, thus, must not be neglected. Our parametrization and calculation should be applied to this kinematical region but not to an experiment with arbitrary, but not necessarily

small $|\vec{q}\,|$, which is the main application of the multipole expansion[2].

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